Monitoring Of A Heterogeneous Process Using Broadband Acoustic Emission Measurements

Adam Besenyei, Leonid Gershuni , Ignacio Plans,
Maria Rechberger, Olga Shipilova

Instructor: Tony Mulholland

October 11, 2004

Abstract

The paper is devoted to mathematical modelling of the problem of broadband acoustic emission measurements. In reality the acoustic emission depends on a number of parameters. To develop the mathematical model of this complex process assumptions, simplifying the problem, are done. The one-dimensional model for one particle is presented as a result of the first stage of modelling. The model constructed is used to simulate the measurement of acid particle in toluene. Numerical results obtained are presented.

1 Introduction

Competition in the world-wide markets makes high technology companies reduce the costs and time required to market a product and simultaneously, improve the quality and consistency of manufacture. This means that manufacturing processes need to be optimized and controlled efficiently in a responsive manner. To meet this requirement, there is a need for acquisition of physical and chemical information in real time. Variation of both factors affects product consistency, although it is often variation in the physical properties of materials that causes the most problems in terms of downstream processing. For example, changes in particle size distribution can result in segregation and sticking problems in the secondary manufacturing processes of blending and tablet pressing, respectively, which may mean that entire batches have to be reworked or scrapped.

The ability to monitor and control processes better will result in a reduction in production time and costs, and the operation of safer, more consistent and higher quality manufacturing processes. It is estimated that such improvements could result in an increase in profits of up to 20 percent for manufacturers of high-value products such as pharmaceutical companies. For example, the ability to manufacture the active and excipient components, which make up a tablet, to a specified particle size and size distribution with less variability would reduce the number of batch failures owing to poor blend uniformity. The estimated cost of a failed pharma batch is \$0.25 million. Hence, the development of improved acoustic techniques, which are relatively low cost in comparison to many optical techniques, is desirable for more effective process investigation, monitoring and control.

Acoustic emission, according to ASTM[1], refers to the generation of transient elastic waves during the rapid release of energy from localized sources within a material. The source of these emissions, for instance, in metals is closely associated with the dislocation movement accompanying plastic deformation and the initiation and extension of cracks in a structure under stress. Other sources of acoustic emission are melting, phase transformation, thermal stresses, cool down cracking and stress build up.

The acoustic emission technique is based on the detection and conversion of these high frequency elastic waves to electrical signals. This is accomplished by directly coupling piezoelectric transducers on the surface of the structure under test and loading the structure. The output of each piezoelectric sensor (during structure loading) is amplified through a low- noise preamplifier, filtered to remove any extraneous noise and furthered processed by suitable electronic equipment, for instance, an oscilloscope. The signals obtained are undergone by Fourier transform to receive the packet of the main wave frequency. This packet has higher information content.

The acoustic emission technique has a number of advantages, such as:

- the technique is non-invasive, non-destructive and relatively inexpensive;
- the process gives information on physical properties even for opaque samples analyzed;
- no window required;
- multi-point and real-time measurements can be made;
- the technique can early and rapid detection of defects, flaws, cracks etc.

Many codes and standards exist for acoustic emission testing of vessels, from transportation gas cylinders and railroad tanks to thousands tons storage tanks. The model system for investigation physical parameters, influencing acoustic emission, usually consists of a vessel with an oil jacket, stirrer, creates mechanical perturbation in substance tested, and a transducer, recording elastic waves and transforming then into electrical signal. Having done computer treating, one obtains a spectrogram.

Being an interesting physical and chemical problem, the acoustic emission presents a good example of real-life problems for mathematical modelling. From this point of view the problem of acoustic emission measurements can be divided into three subproblems: modelling movement of one or more particles within a vessel; modelling particle impact to a membrane and its vibration produced, and modelling a transducer.

The rest of the paper is organized as follows. A mathematical model developed is formulated in Section 2 with detailed description of each part. In Section 3, two computational experiments are presented. Some conclusions are drawn in Section 4.

2 Mathematical Model of Acoustic Emission

The physical model of an acoustic emission reactor is presented in Fig.1. A small amount of the solid substance (particles) to be tested is mixed with a fluid within a vessel, having an

[1]American Society for Testing and Materials,http://www.astm.org

Figure 1: Experimental setting

oil jacket. The solution is perturbed by a stirrer. The mechanical perturbations obtained are recorded by a transducer, which transforms them into electrical waves. Obviously, the acoustic emission depends on a number of parameters. The frequency of particles impacts is determined by the number, material, concentration and size of particles, the rate of stirrer rotation, material and geometry of the vessel, the properties of fluid. The recording of the mechanical waves is influenced by sensitivity of the transducer.

To develop the mathematical model of this complex process a number of assumptions are done. Firstly the problem is reduced to one dimension. It is supposed there is only one particle within the 1D vessel. The physical and chemical properties of the particle, fluid and vessel are assumed to be constant. The stirrer rate is a problem parameter. It is assumed that the particle impact to the transducer membrane is non-plastic.

The model of the acoustic emission reactor is constructed by three submodels:

Model A Particle Movement:

$\star$*input* - the particle characteristic size a, e.g., the diameter;

$\star$*output* - the particle velocity $\mathbf{v}$ in the impact instant;

Model B Impact/Oscillation Simulation:

$\star$*input* - the particle velocity $\mathbf{v}$;

$\star$*output*- the force f;

Model C Transducer Simulation:

$\star$*input* - the force f;

$\star$*output* - the voltage or sensitivity.

Thus, the input parameter for the problem is a characteristic size of the particle. In this paper we suppose that the particle is a circle with diameter a. The output of the mathematical model is the voltage. Further each submodel is considered.

2.1 Model A - Particle Movement

Let the function $x = x(t)$ define distance of the particle from the vessel center, where the stirrer is situated. It is assumed two forces influence the particle movement: *centrifugal* and *drag* forces. In the laminar-flow region this leads to the following equation:

$$m_e \frac{\partial^2 x}{\partial t^2} = m_e \omega^2 x - 3\pi a \eta \frac{\partial x}{\partial t}, \tag{1}$$

where ω is the rotation rate, η is the viscosity coefficient of the fluid, a is the spherical diameter of the particle, m_e is an effective mass of the particle, defined by:

$$m_e = \frac{\pi}{6} a^3 (\rho_s - \rho_f), \tag{2}$$

where ρ_s, ρ_f are density of particle and fluid, respectively. Eq.(1) and (2) present the model of Stokes' diameter determination [1].

The model (1),(2) describes an infinity vessel, i.e. the vessel without walls. It can be complicated by introducing an error term into the drag function, considering the finite extend of fluid [1]. Thus the drag on a sphere is given by:

$$F = 3\pi a \eta \left(1 + \frac{ka}{R - x}\right) \frac{\partial x}{\partial t}, \tag{3}$$

where k is the geometrical constant, equaling 2.104 for a sphere, R is the radius of the vessel.

Having substituted Eq.(3) into Eq.(1) and simplified, we have:

$$\frac{\partial^2 x}{\partial t^2} = \omega^2 x - \frac{18\eta}{a^2(\rho_s - \rho_f)} \left(1 + \frac{ka}{R - x}\right) \frac{\partial x}{\partial t} \tag{4}$$

Both Eq.(1) and Eq.(4) must be simplified by initial and boundary conditions:

$$x|_{t=0} = r_0, \tag{5}$$

$$\frac{\partial x}{\partial t}|_{t=0} = 0, \tag{6}$$

Eq.(1) is an ordinary differential equation of the second order and problem (1),(5),(6) has the analytical solution [2]. Its solution presents a damping wave. Problem (4),(5),(6) does not have analytical solution and can be solved by only approximate numerical methods.

2.2 Model B - Impact/Oscillation Simulation

This part of the proposed mathematical model is the submodel of the particle impact with a transducer membrane and following oscillations.

2.2.1 Particle impact

Our model based on famous Hertz theory, which was developed in the end of 19th century, but still used today. This theory was conducted by a number of experiments. Especially the theory is used for non-plastic collisions, for example, collisions of two glass balls. Hertz suggested to consider the problem of impact for two bodies as an equivalent problem in electrostatics. According to the Goldsmith [5] we can approximate the static compression of two isotropic elastic bodies by two paraboloid in the vicinity of the contact line. We assume also that surfaces of the body are perfectly smooth. It is worse to say, that the last approach is inapplicable only for discontinuous surfaces, for example, cones and wedges.

The following requirements for equilibrium of two bodies are used in the Hertz contact theory :

- the total resisting force supplied by vertical component of the pressure with the contact area is equal to the total force applied;
- while the distance from the contact area increasing, the stresses decrease rapidly;
- the components of the displacement are zero at infinity, so we can neglect these displacements far from the contact (i.e. the distance is more than the contact area);
- normal stresses outside the region of the contact must vanish and normal stresses acting on both bodies must balance within the contact area.

The impact of the particle and the vessel's wall causes oscillations.

2.2.2 Circular plate vibrations after the particle impact

This section is summarized as follows:

- Our problem consists of a particle (striker) of mass m_1 with incoming speed $v = U_0$, that impacts on a clamped circular plate, which is initially undeformed.
- Taking advantages of the Hertz theory [9], we calculate the duration of the impact, T_H.
- We propose an impact force, Eq.(14), as a half period sine pulse $F(r,t)$, which mainly lasts a time interval T_H.
- We substitute the obtained force in a normal mode decomposition of the displacement response of the plate, Eq.(8), in order to obtain the final solution for the displacement that we are looking for, Eq.(15).

The displacement, the velocity and the acceleration of a point in the surface of the plate are mechanical input data needed by the transducer to provide the final output voltage. We solve numerically all the equations presented in this section that are needed to calculate the final displacement, Eq.(15). This section is based on the work presented by Akay and Latcha in [7].

2.2.3 Normal mode analysis for plate vibrations

The vibration response of a clamped circular plate to an impact force $u = u(r, \eta, t)$ can be found by solving the classical equation of the plate motion:

$$D(1 + j\eta)\nabla^4 u(r, \theta, t) + \rho h \frac{\partial^2 u(r, \theta, t)}{\partial t^2} = F(r, \theta, t), \quad (7)$$

where $u(r, \theta, t)$ is the displacement of the plate at a point (r, θ) and $F(r, \theta, t)$ is the applied force per unit area of the plate. The constants in Eq. (7) are $D = Eh^3/12(1 - \nu^2)$, which is the flexural rigidity, ρ is the density, h is the thickness, E is the Young's modulus, ν is the Poisson's ratio, and η is the internal loss factor of the plate.

The displacement response of a circular plate, initially undeformed and at rest, with respect to an arbitrary force $F(r, \theta, t)$ can be written as:

$$u(r,\theta,t) = \frac{1}{\rho h}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{\phi_{nm}(r,\theta)}{\omega_{nm}(1+j\eta)^{\frac{1}{2}}}\int_0^t F_{nm}(\tau) sin\left[\omega_{nm}(1+j\eta)^{\frac{1}{2}}(t-\tau)\right]d\tau, \quad (8)$$

where ω_{nm} are the natural frequencies of the plate, n, m denote the radial and circular modes, respectively. The normal mode shapes of the plate, ϕ_{nm}, are determined by applying the boundary conditions to the to the homogeneous, undamped equation of the palte motion. For the axis-symmetric vibrations of a circular plate of the radius a, the normalized mode shapes are given by

$$\phi_n(r) = \frac{1}{\sqrt{2}}\left(\frac{J_0(\lambda_n r/a)}{J_0(\lambda_n)} - \frac{I_0(\lambda_n r/a)}{I_0(\lambda_n)}\right), \quad (9)$$

where λ_n are the roots of the frequency equation:

$$J_0(\lambda_n)I_1(\lambda_n) + J_1(\lambda_n)I_0(\lambda_n) = 0.$$

J_0 and J_1 are Bessel functions of the first kind and I_0 and I_1 are modified Bessel functions of the first kind. The natural frequencies of the plate ω_n are given by:

$$\omega_n = (\lambda_n/a)^2(D/\rho h)^{\frac{1}{2}}. \quad (10)$$

2.2.4 The impact force

The expression for the contact force developed during elastic impact of the spherical striker of the radius a with the rigid plane surface of a semi-infinite solid has been given by Hunter in [8] as a function of a relative approach α of the striker and the impacted plane surface:

$$F(t) = ka(t)^{\frac{3}{2}}, \quad (11)$$

where

$$k = \frac{4}{3}\sqrt{b}\left(\frac{(1-\nu_1^2)}{E_1} - \frac{(1-\nu)^2}{E}\right)^{-1}.$$

Here ν_1, ν and E_1, E are the Poisson's ratios and the elasticity moduli of the sphere and the impacted object, respectively. The contact-force expression given in Eq.(11) is an extension of the well-known Hertz contact theory developed for the static contact curved bodies [9].

In the case of impact of a sphere with a thin plate, the force-time history is found by combining Eq.(11) with the equation of the palte motion Eq.(7). The result nonlinear differential equation obtained by Zener in [10] for the case of a large plate (where the reflections from the boundaries of the plate return to the impact region after the contact ceases) is given by:

$$F\frac{d^2f}{dt^2} - \frac{1}{3}\left(\frac{dF}{dt}\right)^2 + \frac{3}{2}k^{\frac{3}{2}}\left(\frac{1}{m_1}F^{\frac{7}{3}} + \frac{3(1-\nu^2)}{4h^2\rho E}F^{\frac{4}{3}}\frac{dF}{Dt}\right) = 0. \quad (12)$$

According to [10], this equation is adimensionalized to obtain:

$$\frac{d^2\sigma}{d\tau^2} + \left(1 + \lambda\frac{d}{d\tau}\right)\sigma^{\frac{3}{2}} = 0, \tag{13}$$

where $\tau = t/T$, $\sigma = \alpha/(TU_0)$, and $T = 0.311T_H$. Here T_H is the duration of the impact predicted by the Hertz theory for infinitely rigid surfaces and it is given by

$$T_H = 2.9432(\alpha_m/U_0).$$

The maximum value of the relative displacement α is expressed by:

$$\alpha_m = [5U_0^2 m_1/(4k)]^{2/5},$$

where U_0 is the impact velocity.

Eq.(13) depends on a nondimensional constant λ, referred to as *the inelasticity parameter*, which is equal to $3.218m_1/(T_H Z)$, where m_1 is the mass of the striker and $Z = 8(D\rho h)^{1/2}$ is the impedance of the plate at the impact position.

The force-time history obtained in [7] from Eq.(13) varies with the height and the width, and depends on the values of the inelasticity parameter λ. In cases of very small λ , the contact time history resembles the square of half-period sine wave. As λ increases, the impact force amplitude decreases and the duration of contact is longer in the second half of the contact period. The maximum value of the normalized impact force decays exponentially with λ. Since the inelasticity parameter λ is inversely proportional to the second power of plate thickness h, for a given impact velocity the impact force amplitude increases nonlinearly with plate thickness and mass of the striker, reaching its maximum value predicted by the Hertz theory of a plate with the semi-infinite thickness.

In our work the force developed during the impact between a spherical impactor and a plate is represented as a point force with a squared half-period sine-wave time history, see, e.g., [7]:

$$F(r,t) = F_0\delta(r)\sin^2\omega_0 t/2\pi r, \quad 0 \le t \le \pi/\omega_0, \tag{14}$$

where the duration of contact $T_H = \pi/\omega_0$ is obtained from the Hertz theory and the amplitude F_0 is obtained by using the inelasticity parameter, according to the analysis described above.

2.3 Response of the plate to the impact

The axis-symmetric displacement response of a circular plate with clamped outer edge to an impact force by a a spherical object can by found by evaluating the response in Eq. (8) with the forcing function in Eq. (14), which gives:

$$u(r,t) = \frac{F_0}{M}\sum_{n=0}^{\infty}\frac{\phi_n(0)\phi_n(r)}{(4\omega_0^2 - \omega_n^2\sin\Omega - 2\eta\omega_n\omega_0\cos\Omega} \cdot \{\}, \tag{15}$$

where $\{\}$ stands for

$$\{\} \to \begin{cases} \frac{1}{2}\sin(2\omega_0 t + \Omega) + X_1 e^{-(\eta/2\omega_n)t}\sin(\omega_n^* t + \Omega_1) + E & 0 \le t \le \pi/\omega_0 \\ X_2 e^{(-\eta/2)\omega_n t_1}\sin(\omega_n^* t_1 + \Omega_2) & t_1 = t - \pi/\omega_0 \ge 0. \end{cases}$$

The constants are given as follows:

$$\begin{cases} \Omega = \tan^{-1}\left[(\omega_n^2 - 4\omega_0^2)/2\eta\omega_0\omega_n\right], \\ \Omega_1 = \tan^{-1}\left(\frac{\omega_n^*(E+\frac{1}{2}\sin\Omega)}{\omega_0\cos\Omega+\frac{1}{2}\eta\omega_n(E+\frac{1}{2}\sin\Omega)}\right), \\ \Omega_2 = tan^{-1}\left[C_1\omega_n^*/(C_2 + \frac{1}{2}C_1\eta\omega_n)\right], \\ X_1 = \omega_0\cos\Omega/\left(\frac{1}{2}\eta\omega_n\sin\Omega_1 - \omega_n^*\cos\Omega_1\right), \\ X_2 = C_2/\left(\omega_n^*\cos\Omega_2 - \frac{1}{2}\eta\omega_n\sin\Omega_2\right), \\ C_1 = \frac{1}{2}\sin\Omega + X_1\exp(-\eta\omega_n\pi/2\omega_0)\sin(\omega^*\pi/\omega_0 + \Omega_1) + E, \\ C_2 = \omega_0\cos\Omega + X_1\exp(-\eta\omega_n\pi/2\omega_0)\left[\omega_n^*\cos(\omega_n^*\pi/\omega_0 + \Omega_1) - (\eta/2)\omega_n\sin(\omega_n^*\pi/\omega_0 + \Omega_1)\right], \\ E = \left[(4\omega_n^0 - \omega_n^2)\sin\Omega - 2\eta\omega_0\omega_n\cos\Omega\right]/2\omega_n^2, \\ M = \rho\pi a^2 h, \\ \omega_n^* = \omega_n(1-\eta^2/4)^{1/2}. \end{cases}$$

The velocity and acceleration of the plate are obtained by differenciating Eq.(15) once and twice, respectively. We develope these operations numerically.

2.4 Model C - Transducer Simulation

In this part we make a simple model of a piezoelectric transducer. Roughly speaking transducers can convert mechanical energy to electrical energy and conversely. More precisely in transmission mode applying a voltage on the transducer leads to mechanical energy and a pressure wave being emitted. Conversely, in receiving mode a mechanical impulse, for instance, a stress wave, can be converted to electrical energy and then to an electrical signal. Naturally, in passive acoustics we investigate only receiving mode transducers. The simplest example of a transducer is a human ear. The voice waves come to our ear and it is converted to an electrical signal, so that we can hear the voice. Now let us consider a monolithic piezoelectric plate transducer. It is made of a piezoelectric material with a backing material and a front matching layer. For this type of transducer a simple one-dimensional model can be derived.

We summarize the derivation of the main characteristic of the transducer called reception sensitivity, see, e.g., [6].

We have three starting equations. The first two are the piezoelectric equations:

$$T = YS - hD, \tag{16}$$
$$E = -hS + \frac{D}{\varepsilon}, \tag{17}$$

where T is the stress, Y is the Young's modulus, S is the strain, h is the piezoelectric constant, D is the electrical displacement, E is the electric field and ε is the permittivity.

We also need the one-dimensional wave equation for mechanical displacement:

$$\rho\frac{\partial^2\xi}{\partial t^2} = Y\frac{\partial^2\xi}{\partial x^2}, \tag{18}$$

where ρ is the density and ξ is the displacement. Due to the clamped capacitance C_0 of the transducer we have boundary conditions at $x = 0$ and $x = L$ where L is the thickness of the transducer. By taking the Laplace transform of (18) and solving it we get (with constants of integration A, B)

$$p^2\bar{\xi} = v^2 \frac{\partial^2 \bar{\xi}}{\partial x^2}, \tag{19}$$

where the wave speed v is given by $v^2 = Y/\rho$.

We know that the voltage is the integral of the electric field and the force at the front face of the transducer F is given by $\bar{F} = A_r \bar{T}$, where A_r is the cross-sectional area of the transducer. Now using (17) and (19) we get the following for the voltage

$$\bar{V} = -h[A(e^{-p\tau} - 1) + B(e^{p\tau} - 1)]\bar{U}$$

where $\tau = L/v$ is the transit time of a plane wave through the transducer and $\bar{U} = p\,C_0 \bar{Z}_E/(1 + p\,C_0 \bar{Z}_E)$. Here $\bar{Z}_E$ is the electrical impedance of an arbitrary electrical load.

Before going on we make some assumptions on the backing material and the fronting material. We require differentiability of the displacement ξ at the interfaces of these materials, which is the same as the continuity of force. Let us denote Z_c, Z_1, Z_2 respectively the mechanical impedance of the transducer, the backing and the fronting material. With these we can introduce T_F, T_B, R_F, R_B defined by

$$T_F = \frac{2Z_c}{Z_c + Z_1}, \quad T_B = \frac{2Z_c}{Z_c + Z_2},$$

$$R_F = \frac{Z_c - Z_1}{Z_c + Z_1}, \quad R_B = \frac{Z_c - Z_2}{Z_c + Z_2},$$

which are the front and back transmission coefficients, and the front and back reflection coefficients respectively. Then after some transformations and simplifications by using the differentiability conditions, see [6], one can get

$$\frac{\bar{V}}{\bar{F}} = \frac{-hT_F \bar{K}_F \bar{U}/pZ_c}{1 - h^2(\bar{K}_F T_F + \bar{K}_B T_B)\bar{U}/(2p^2 Z_c \bar{Z}_E)}. \tag{20}$$

Here $\bar{K}_F$ and $\bar{K}_B$ are given by

$$\bar{K}_F = \frac{(1 - e^{-p\tau})(1 - R_B e^{-p\tau})}{1 - R_F R_B e^{-2p\tau}}, \quad \bar{K}_B = \frac{(1 - e^{-p\tau})(1 - R_F e^{-p\tau})}{1 - R_F R_B e^{-2p\tau}}.$$

The left side of (20) is the main characteristic of the transducer called the reception sensitivity. Sensitivity means the received voltage from a unit impulse force. The maxima of the sensitivity show the frequencies that the transducer "hears".

In Fig.2 we can see that our transducer has got the maximum voltage of $3.7 10^{-7}$ V/N at the frequency $\omega = 0.5 10^5$ Hz. This is the transducer's resonant frequency.

3 Results

Model A of the acoustic emission model is the simulation of the particle movement within the vessel. From our assumption, discussed above, we have constructed two mathematical model for particle movement in centrifugal field. Problem 1, consisting of Eq. (1) and initial and boundary conditions (5),(6), has the analytical solution. Problem (4),(5),(6)

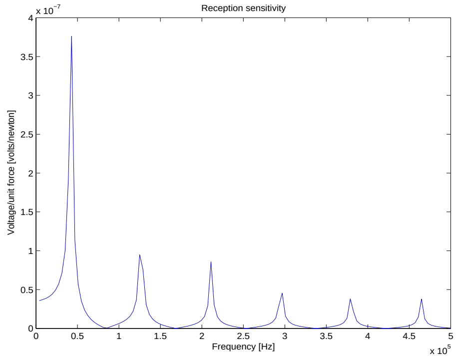


Figure 2: Sensitivity (volts/newtons)10^{-7} with respect to the frequency (Hertz)10^{5} for a piezoelectric transducer. The material properties are in Tab. 1.

Table 1: Transducer Properties

	Constants	Units	Value
Piezoelectric constant	h	Vm^{-1}	2.610^{9}
Capacitance	C_0	F	10^{-10}
Specific Mechanical Impedance	Z_c	$kgm^{-2}s^{-1}$	710^{6}
Density	ρ	kgm^{-3}	7.510^{3}
Transducer thickness	L	m	10^{-2}
Transit time	τ	s	3.2510^{-6}
Backing material impedance	Z_1	$kgm^{-2}s^{-1}$	210^{6}
Front material impedance	Z_2	$kgm^{-2}s^{-1}$	1.510^{6}
Cross-sectional area	A_r	m^{-2}	10^{-4}

can be solved only numerically.

Consider the acoustic reactor with the following parameter: the vessels's radius R equals 0.1; the stirrer velocity ω equals $\frac{7\pi}{36} = 250$rpm; the particle material is acid with density $\rho_s = 1573$; the fluid in the vessel is toluene with density $\rho_f = 865$ and viscocity $\eta = 6.810^{-3}$.

Tab. 2 and Tab. 3 present result velocity depending on the particle size a for the initial condition $r_0 = 0.9R$ and $r_0 = 0.5R$, respectively. The tables show also the time of collision the particle with the vessel's wall. Time 1 and velocity 1 are results of problem 1, time 2 and velocity 2 result from problem 2.

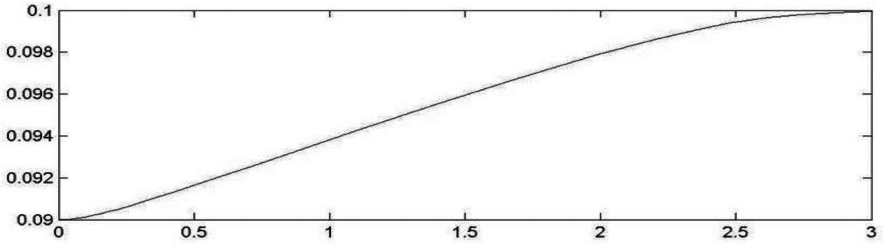
Table 2: Time and velocity of the impact. Initial condition $r_0 = 0.9R$

a	time 1	velocity 1	time 2	velocity 2
0.00005	195.25	5.410^{-5}	214.6866	4.22610^{-7}
0.00015	21.709	4.9510^{-4}	27.984	1.165210^{-6}
0.00025	7.85	0.0013	11.573	1.749310^{-6}
0.0005	21.709	4.9510^{-4}	27.984	1.165210^{-6}

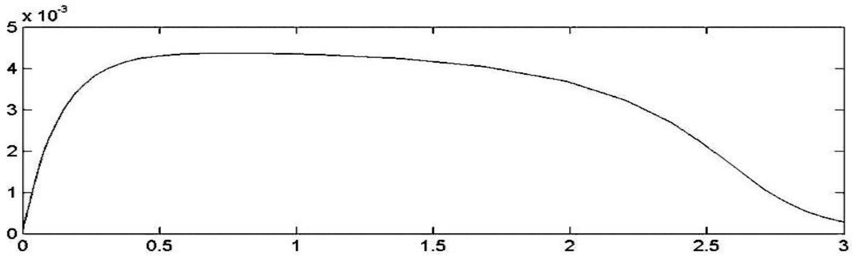
Table 3: Time and velocity of the impact. Initial condition $r_0 = 0.5R$

a	time 1	velocity 1	time 2	velocity 2
0.005	2.2051	0.0508	2.2233	0.0431
0.0005	13.0877	0.054	14.7420	4.9710^{-5}
0.00005	1284.5	5.3910^{-5}	1306	4.52510^{-7}

In Fig.3(a) the infinite solution of problem 1 and in Fig.3(b) the bounded solution of problem 2 are demonstarted: vertical axis is the distance from the vessel center, horizontal one is particle velocity.



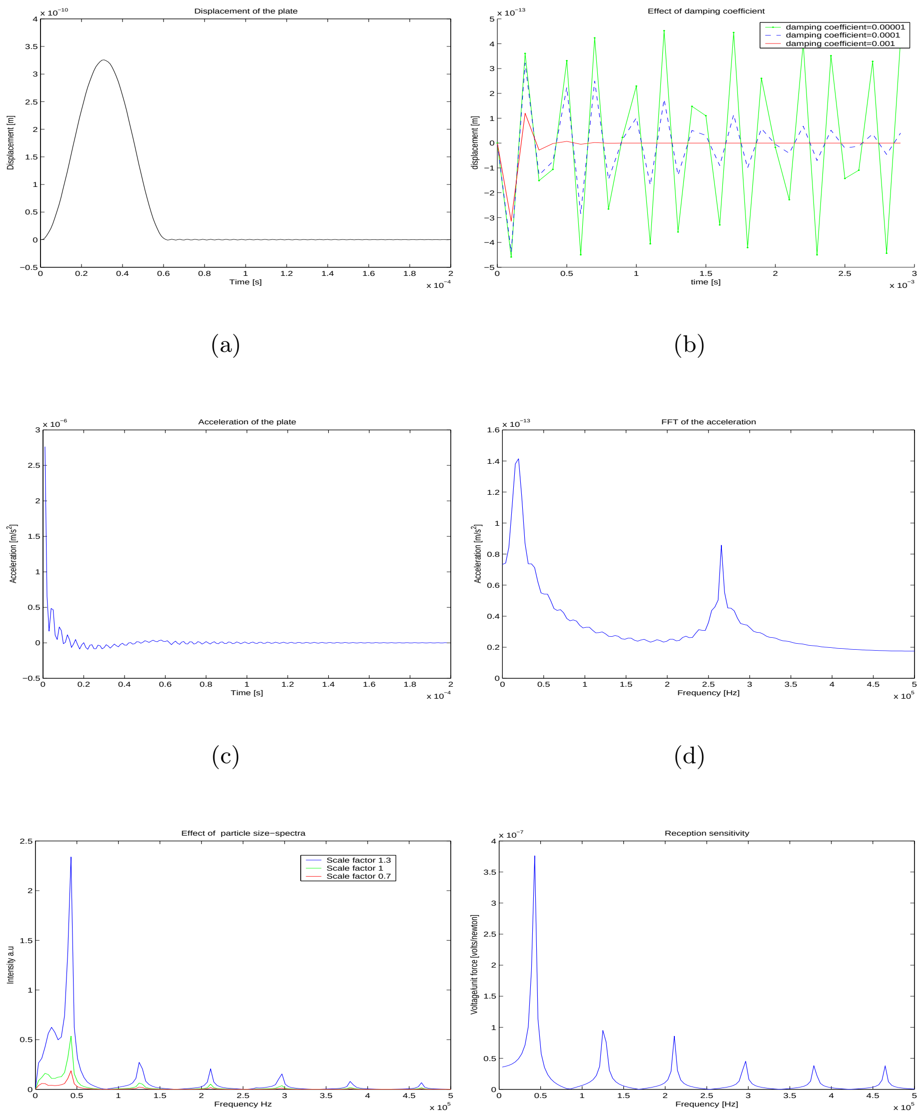
(a)



(b)

Figure 3: (a) - solution of problem 1, "infinite" vessel; (b) - solution of problem 2, "bounded" vessel.

The second part, Model B, of the proposed model has the velocity of the impact as an input parameter. Having solved our system of equations with $v = 0.0508$, we receive the following dependence of the displacement from the time, Fig. 4(a). Main peak corresponds to the period of impact. Displacement depends from the damping coefficient of the plate material is demonstrated in Fig. 4(b). The acceleration of the plate is the second derivative of the plate in each point. In Fig.4(c) one can see the evolution of the acceleration with time. The fast fourier transform of the acceleration is shown in Fig.4(d). The main result of the Model B is the dependence between particle size and intensity, Fig.4(e).



(a) (b) (c) (d) (e) (f)

Figure 4: (a) – the displacement of the plate; (b) – the effect of damping coefficients; (c) – acceleration of the plate; (d) – the FFT acceleration; (e) – the effect of the particle size; (f) – the reception sensitivity of the transducer.

The output data of the presented model is sensitivity of the transducer with respect to voltage for given system parameters. In Fig. 4(f) is depicted the solution with following initial parameters: the vessels's radius R equals 0.1; the stirrer velocity ω equals $\frac{7\pi}{36}$ = 250rpm; the particle size is 0.005; the particle material is acid with density $\rho_s = 1573$; the fluid in the vessel is toluene with density $\rho_f = 865$ and viscosity $\eta = 6.810^{-3}$; and obtained velocity of impact $v = 0.0508$.

4 Conclusion

We have developed the one-dimensional model for the broadband acoustic emission. The proposed model is a sequence of three submodels, such as the motion of a particle inserted in a rotating fluid, the particle's impact force, causing plate vibrations, plate's distortions, resulting in deformation force, and a linear transducer, transforming elastic waves into electrical ones. Thus the solution of our model is voltage patterns associated to particle's properties. The results obtained are not very comparative with experimental measurements, that is explained hard assumption on the mathematical model. In the viewpoint of further work we should consider a multi-particle model.

Acknowledgements

Authors would like to thank organizers of the ECMI modelling week for possibility to take part in this school, simulating the real-life connection between industry and science. Authors appreciate Tony Mulholland for the interesting problem and overall help during the work very much.

References

[1] Allen T. *Particle Size Measurement.* Powder Technology Series, 1974.

[2] Rade L., Westergren B. *Mathematics Handbook for Science and Engineering*, Studentlitteratur, Lund, 2001.

[3] Lee J., Ladd A.J.C. *Axial Segregaion in a Cylindrical Centrifuge*, Physcal Review Letters.

[4] Nordling C., Österman J. *Physics Handbook*, 1980.

[5] Goldsmith W. *Impact: The Theory and physical behaviour of colliding solids*, 1960.

[6] Orr L-A., Mulholland A.J. Thickness Mode Response of a 1-3 Composite Transducer with an SBS Passive Phase, *Research Report No. 8, Department of Mathematics, University of Strathclyde, Glasgow, UK,* 2004.

[7] Akay A., Latcha M. Sound radiation from an impact-excited clamped circular plate in an infinite baffle *J. Acoust. Soc. Am.* 1983, 74(2).

[8] Hunter S. C. Energy absorbed elastic waves during impact, *J. Mech. Phys. Solids* 1957, 5, 162-171.

[9] Love A.E.H. A treatise on the Mathematical Theory of Elasticity. *Dover, New York*, 1944, 4th ed., 198-200.

[10] Zener C. Intrinsic inelasticity of large plates, *Phys. Rev.* 1941,59, 669-673.