Dynamic control of network-based epidemic models

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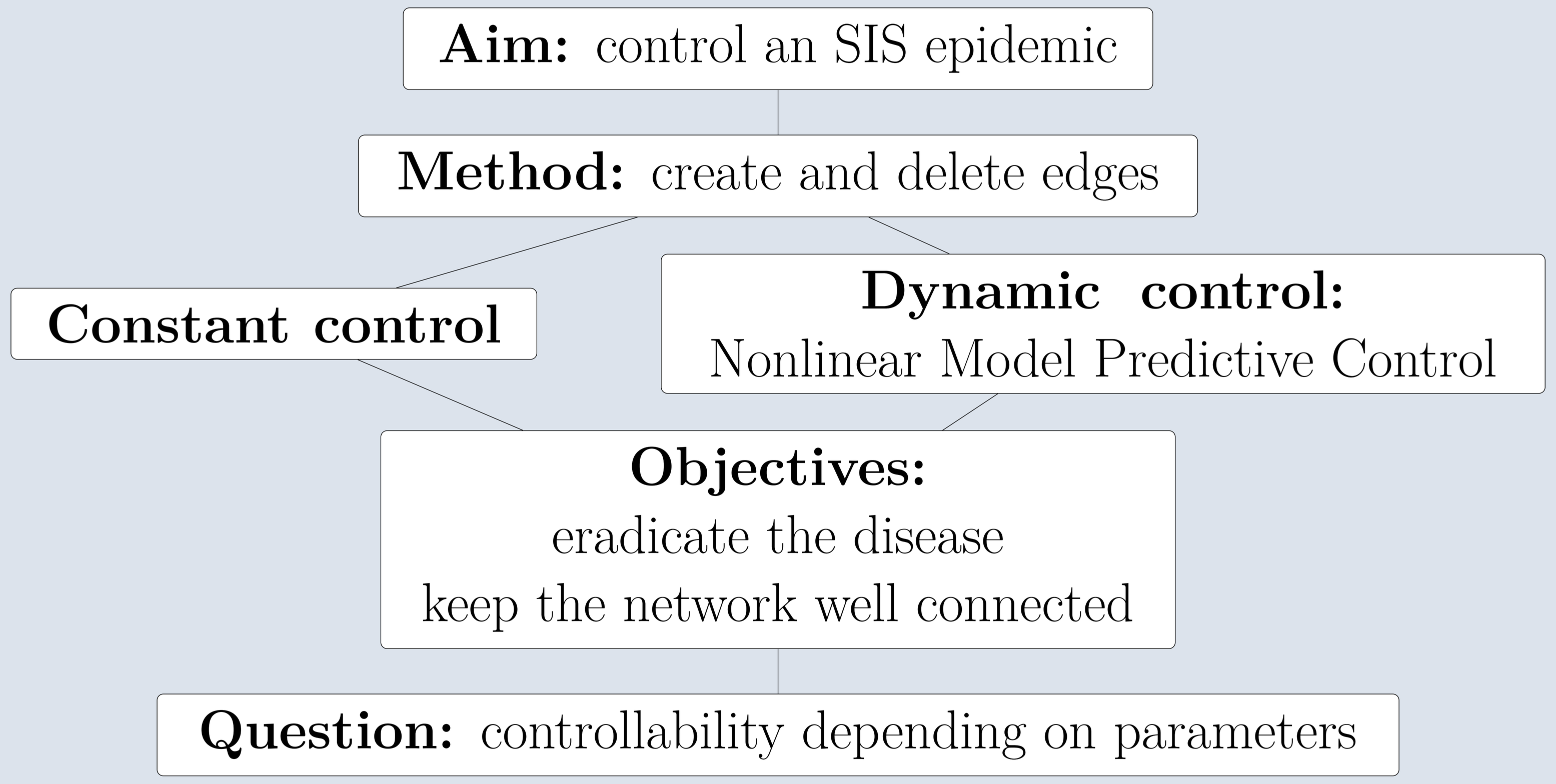
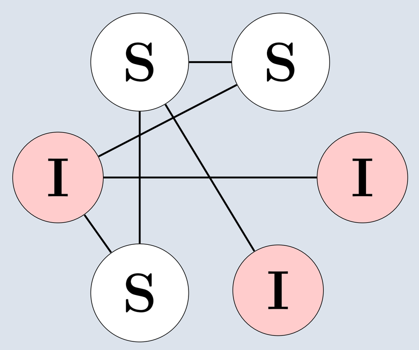
Big picture

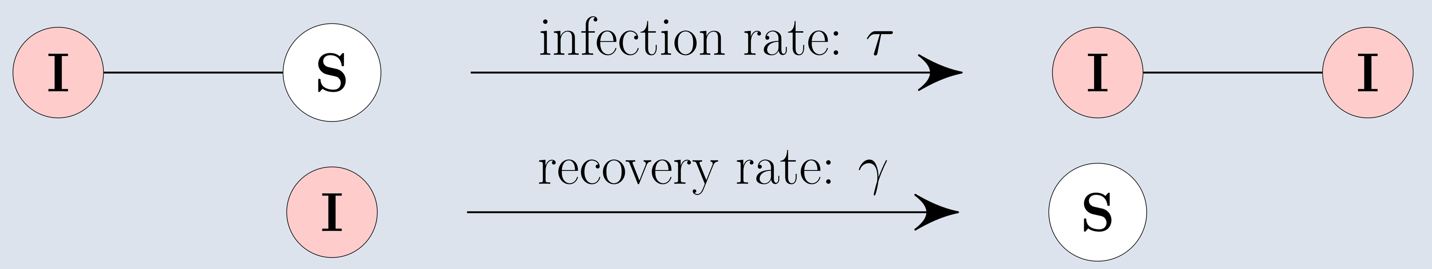


SIS epidemics & control

SIS epidemics: network of N individuals



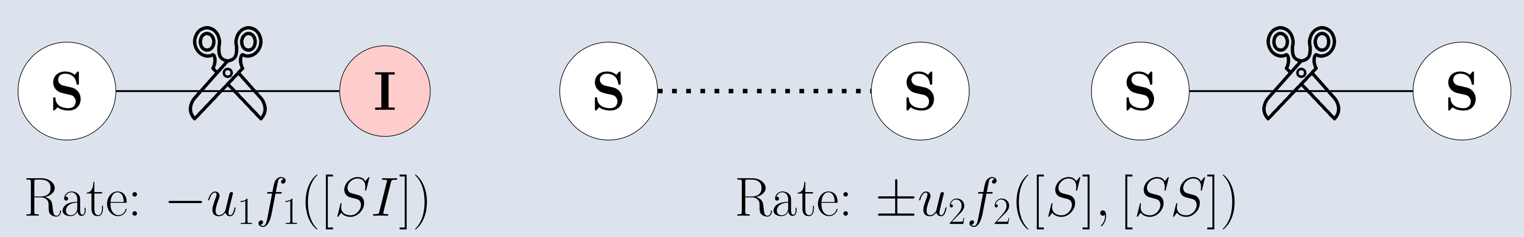
Infection – Recovery:



Variables: expected number of singles and pairs, mean degree

$$[I], [S], [SI], [SS], [II], \quad n(t) = \frac{[SS] + 2[SI] + [II]}{N}.$$

Control:



Classical pairwise model with control:

$$\dot{[I]} = \tau[SI] - \gamma[I],$$
$$\dot{[SI]} = \gamma([II] - [SI]) + \tau([SSI] - [ISI] - [SI]) - u_1 \cdot f_1([SI]),$$
$$\dot{[II]} = -2\gamma[II] + 2\tau([ISI] + [SI]),$$
$$\dot{[SS]} = 2\gamma[SI] - 2\tau[SSI] + u_2 \cdot f_2([S],[SS]).$$

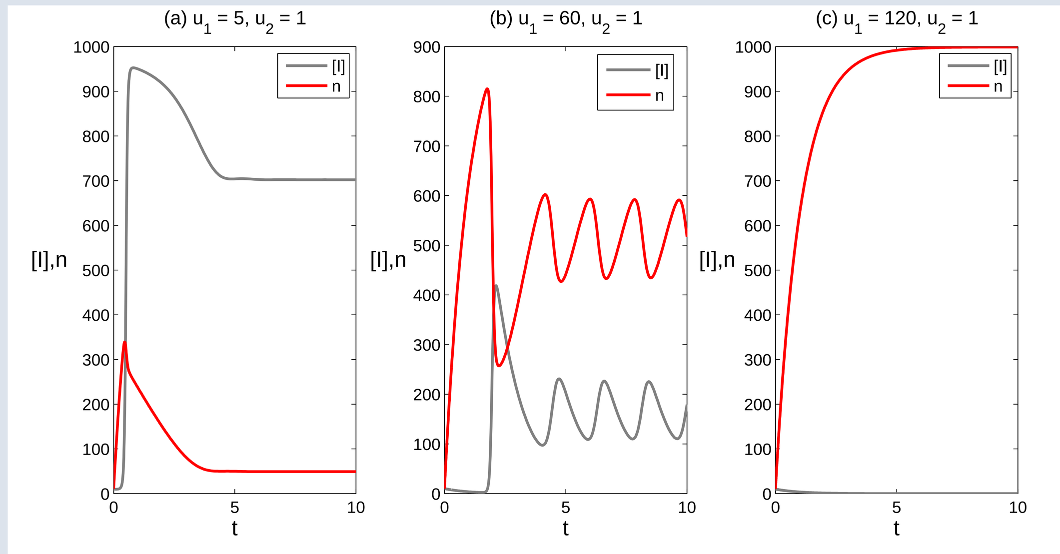
Closures: approximation of $[SSI]$ and $[ISI]$ with pairs

$$[SSI] \approx \frac{n-1}{n} \cdot \frac{[SS][SI]}{N - [I]}, \qquad [ISI] \approx \frac{n-1}{n} \cdot \frac{[SI]^2}{N - [I]}.$$

Desired outcome: $I(T) = 0$, $n(T) = n(0)$, for some $T > 0$.

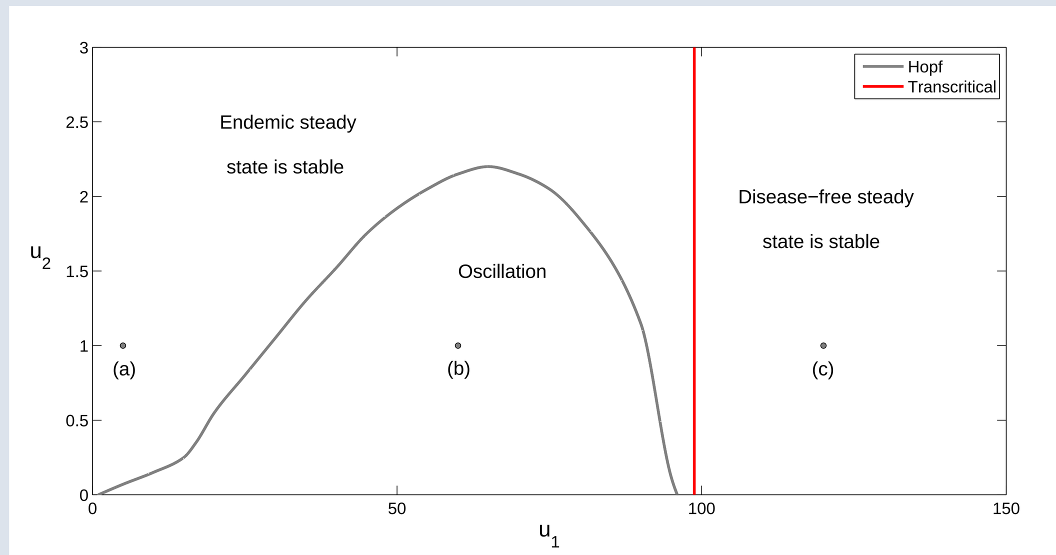
Constant control

Control: $u_1, u_2 \geq 0$, $f_1([SI]) = [SI]$, $f_2([S],[SS]) = [S]([S]-1) - [SS]$.
Behaviours: three cases



$N = 1000$,
$\tau = 0.1$, $\gamma = 1$,
$I(0) = 10$,
$n(0) = 10$.

Bifurcation diagram:



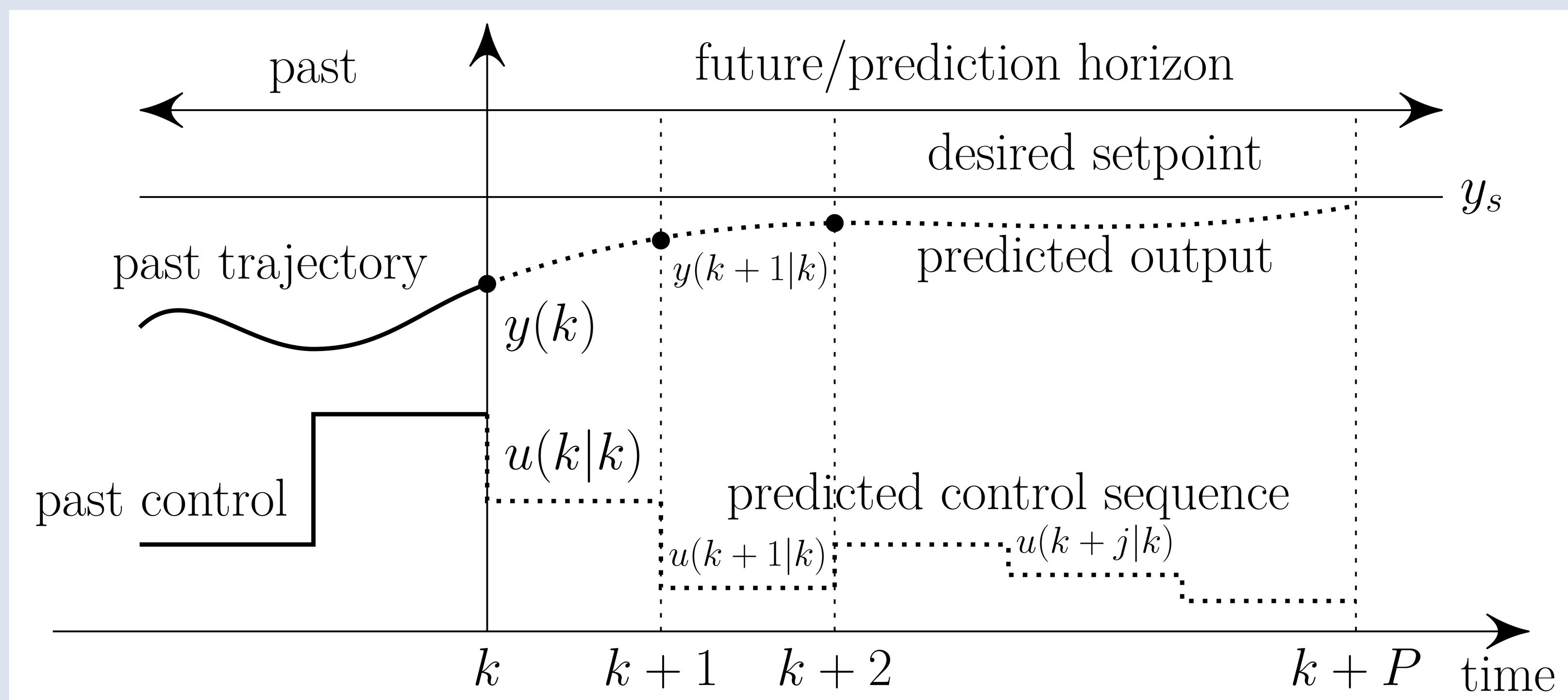
Transcritical bifurcation set:
$u_1 = \tau(N-2) - \gamma$

Conclusion: eradication possible but network becomes complete

NMPC

Discrete time system: observations at $k\Delta t, k \in \mathbb{Z}$

$$\begin{cases} x(k+1) = F(x(k), u(k)), & \text{where } x \text{ state, } u \text{ control variable,} \\ y(k+1) = h(x(k+1)), & y \text{ output (measured) signal.} \end{cases}$$



Optimal control signal: nonlinear optimization (Matlab: lsqnonlin)

$$\underset{u(k|k),\dots,u(k+P|k)}{\arg\min} \sum_{j=0}^{P} \lambda_1\big(y(k+j|k) - y_s\big)^2 + \lambda_2\big(u(k+j|k) - u(k+j-1|k)\big)^2$$

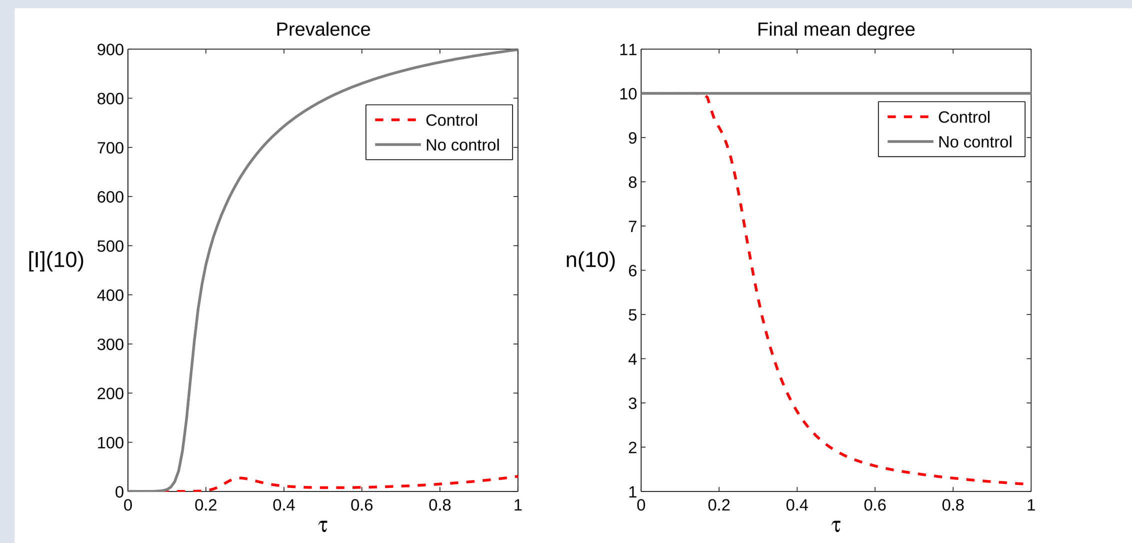
Moving Horizon: only $u(k|k)$ is applied, then the prediction horizon is moved one step forward and the same procedure is performed.

Dynamic control with NMPC

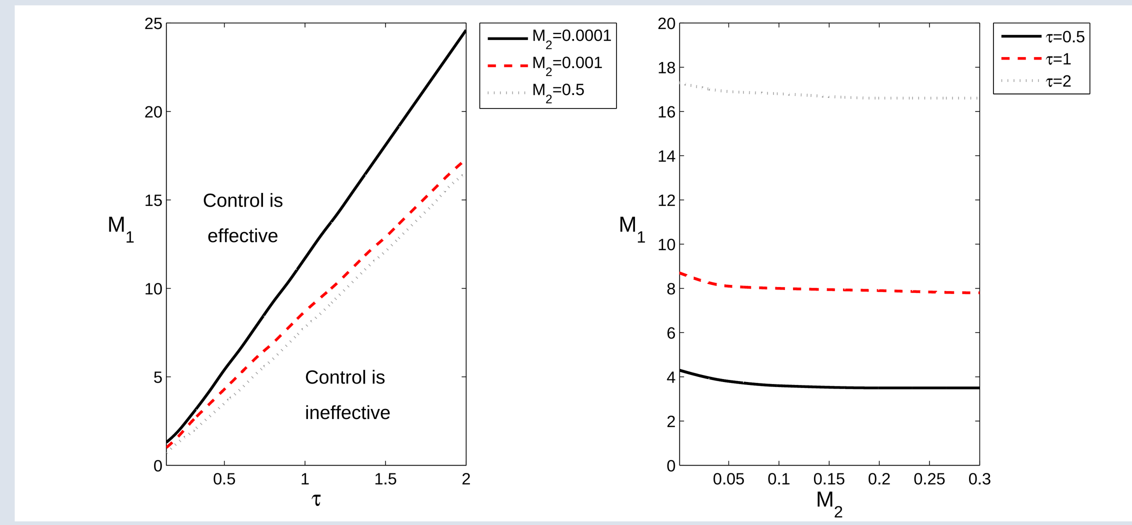
Control: $0 \leq u_1 \leq M_1, |u_2| \leq M_2$ piecewise constant, $f_1([SI]) = [SI]$, $f_2([S],[SS]) = \max\{u_2, 0\} \cdot ([S]([S]-1) - [SS]) + \min\{u_2, 0\} \cdot [SS]$.
Parameters:

N	population size	1000
γ	recovery rate	0.1
$I(0)$	initial infected population size	10 (1%)
$n(0)$	initial mean degree	10
T	time to end of control	10 $(10 \cdot 1/\gamma)$

Interplay between τ and M_1:

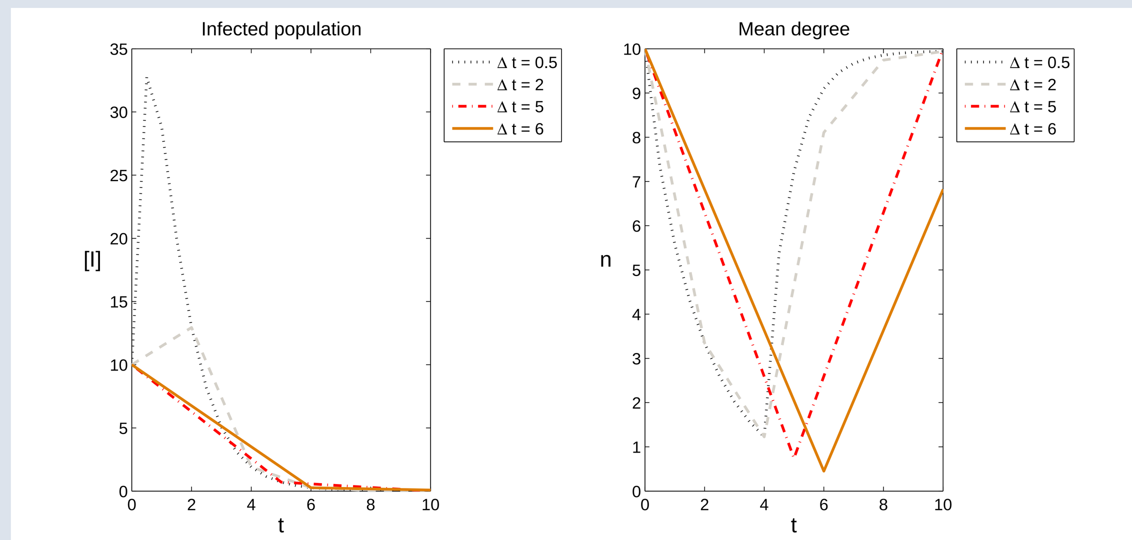


$\tau = 0.1$,
$\Delta t = 0.1$,
$M_1 = 1$,
$M_2 = 0.001$.



$\Delta t = 0.1$.

Effect of Δt:



$\tau = 1$,
$M_1 = 7.8$,
$M_2 = 0.5$.

Further possible questions: dependence on damping parameters of J, adjustment of error term $|[I](T) - 0| \leq \varepsilon$, $|n(T) - n(0)| \leq \varepsilon$.

References

[1] L. Grüne, J. Pannek, *Nonlinear Model Predictive Control*, Springer-Verlag, London, 2011.

[2] Fanni Sélley, Ádám Besenyei, Istvan Z. Kiss, Péter L. Simon, Dynamic control of modern, network-based epidemic models, 2013.