simple, in fact, that we have been able to solve it completely in a few lines. It is important, therefore, to see if this model, with its simplicity, has any relationship at all with reality. Let $p(t)$ denote the human population of the earth at time t. It was estimated that the earth's human population in 1961 was 3,060,000,000 and that during the past decade the population was increasing at a rate of 2%/yr. Thus $t_0 = 1961$, $p_0 = (3.06)10^9$, and $a = 0.02$, so that

$$p(t) = (3.06)\,10^9\,e^{0.02(t-1961)}.$$

We can certainly check this formula out for past populations.

Result. It reflects with surprising accuracy the population estimate for the period 1700–1961. The population of the earth has been doubling about every 35 years, and our equation predicts a doubling of the earth's population every 34.6 years. To prove this, observe that the human population of the earth doubles in a time $T = t - t_0$ where $e^{0.02T} = 2$. Taking logarithms of both sides of this equation gives $0.02T = \ln 2$ so that $T = 50\ln 2 \simeq 34.6$. However, let us look into the distant future. Our equation predicts that the earth's population will be 200,000 billion in the year 2510, 1,800,000 billion in the year 2635, and 3,600,000 billion in the year 2670. These are astronomical numbers whose significance is difficult to gauge. The total surface of this planet is approximately 1,860,000 billion square feet. Eighty percent of this surface is covered by water. Assuming that we are willing to live on boats as well as land, it is easy to see that by the year 2510 there will be only 9.3 square feet per person; by 2635 each person will have only one square foot on which to stand; and by 2670 we will be standing two deep on each other's shoulders.

It would seem therefore, that this model is unreasonable and should be thrown out. However, we cannot ignore the fact that it offers exceptional agreement in the past. Moreover, we have additional evidence that populations do grow exponentially. Consider the Microtus Arvallis Pall, a small rodent which reproduces very rapidly. We take the unit of time to be a month and assume that the population is increasing at the rate of 40%/mo. If two rodents are present initially at time $t = 0$, then $p(t)$, the number of rodents at time t, satisfies the initial value problem $dp(t)/dt = 0.4p(t)$, $p(0) = 2$. Consequently,

$$p(t) = 2\,e^{0.4t}. \tag{1}$$

Table 1 compares the observed population with the population calculated from (1). As one can see, there is excellent agreement.

Remark. In the case of the Microtus Arvallis Pall, p observed is very accurate since the pregnancy period is three weeks, and the time required for the census taking is considerably less. If the pregnancy period were very short then p observed could not be accurate since many of the pregnant rodents would have given birth before the census was completed.