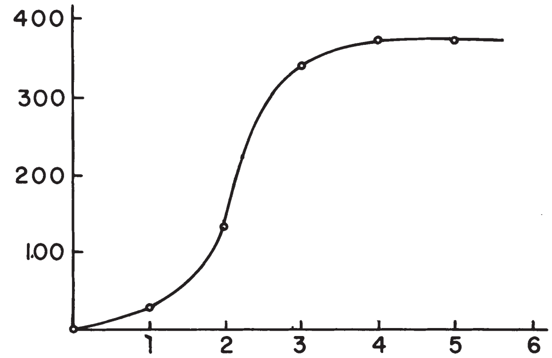

Figure 5.2. Growth of Paramecium

$$0.02 = a - b(3.06)10^9.$$

Consequently, $b = 2.941 \times 10^{-12}$. Thus, according to the logistic law of population growth, the human population of the earth will tend to the limiting value of

$$\frac{a}{b} = \frac{0.029}{2.941 \times 10^{-12}} = 9.86 \text{ billion people.}$$

Note that according to this prediction, we were still on the accelerated growth portion of the logistic curve in 1961, since we had not yet attained half the limiting population predicted for us.

As another verification of the logistic law of population growth, we consider the equation

$$p(t) = \frac{197{,}273{,}000}{1 + e^{-0.03134(t-1913.25)}} \tag{5}$$

which was introduced by Pearl and Reed as a model of the population of the United States [2]. This model was derived in the following manner. First, using the census figures for the years 1790, 1850, and 1910, Pearl and Reed found from (3) that $a = 0.03134$ and $b = (1.5887)10^{-10}$ (see Exercise 2a). Then, to simplify (3), they calculated that the population of the United States achieved half its limiting population of $a/b = 197{,}273{,}000$ in April 1913 (see Exercise 2b). Consequently (see Exercise 2c), we can rewrite (3) in the simpler form (5).

Table 2 compares Pearl and Reed's predictions with the observed values of the population of the United States. These results are remarkable, especially since we have not taken into account the large waves of immigration into the United States and the fact that the United States was involved in five wars during this period.

In 1845 Verhulst prophesied a maximum population for Belgium of 6,600,000, and a maximum population for France of 40,000,000. The

Table 2. Population of U.S. from 1790 to 1950

Year	Actual	Predicted	Error	Percent
1790	3,929,000	3,929,000	0	0.0
1800	5,308,000	5,336,000	28,000	0.5
1810	7,240,000	7,228,000	−12,000	−0.2
1820	9,638,000	9,757,000	119,000	1.2
1830	12,866,000	13,109,000	243,000	1.9
1840	17,069,000	17,506,000	437,000	2.6
1850	23,192,000	23,192,000	0	0.0
1860	31,443,000	30,412,000	−1,031,000	−3.3
1870	38,558,000	39,372,000	814,000	2.1
1880	50,156,000	50,177,000	21,000	0.0
1890	62,948,000	62,769,000	−179,000	−0.3
1900	75,995,000	76,870,000	875,000	1.2
1910	91,972,000	91,972,000	0	0.0
1920	105,711,000	107,559,000	1,848,000	1.7
1930	122,775,000	123,124,000	349,000	0.3
1940	131,669,000	136,653,000	4,984,000	3.8
1950	150,697,000	149,053,000	−1,644,000	−1.1

(The last four entries were added by the Dartmouth College Writing Group)

population of Belgium in 1930 was already 8,092,000. This large discrepancy would seem to indicate that the logistic law of population growth is very inaccurate, at least as far as the population of Belgium is concerned. However, this discrepancy can be explained by the astonishing rise of industry in Belgium and by the acquisition of the Congo which secured for the country sufficient additional wealth to support the extra population. Thus, after the acquisition of the Congo, and the astonishing rise of industry in Belgium, Verhulst should have lowered the vital coefficient b.

On the other hand, the population of France in 1930 was in remarkable agreement with Verhulst's forecast. Indeed, we can now answer the following tantalizing paradox: why was the population of France increasing extremely slowly in 1930 while the French population of Canada was increasing very rapidly? After all, they are the same people! The answer to this paradox, of course, is that the population of France in 1930 was very near its limiting value and thus was far into the period of diminishing growth, while the population of Canada in 1930 was still in the period of accelerated growth.

Remarks.

1. Clearly, technological developments, pollution considerations, and sociological trends have significant influence on the vital coefficients a and b. Therefore, they must be reevaluated every few years.
2. To derive more accurate models for population growth, we should not