Preface to the Second Russian Edition

In the mid-twentieth century the theory of partial differential equations was considered the summit of mathematics, both because of the difficulty and significance of the problems it solved and because it came into existence later than most areas of mathematics.

Nowadays many are inclined to look disparagingly at this remarkable area of mathematics as an old-fashioned art of juggling inequalities or as a testing ground for applications of functional analysis. Courses in this subject have even disappeared from the obligatory program of many universities (for example, in Paris). Moreover, such remarkable textbooks as the classical three-volume work of Goursat have been removed as superfluous from the library of the University of Paris-7 (and only through my own intervention was it possible to save them, along with the lectures of Klein, Picard, Hermite, Darboux, Jordan, ...).

The cause of this degeneration of an important general mathematical theory into an endless stream of papers bearing titles like "On a property of a solution of a boundary-value problem for an equation" is most likely the attempt to create a unified, all-encompassing, superabstract "theory of everything."

The principal source of partial differential equations is found in the continuous-medium models of mathematical and theoretical physics. Attempts to extend the remarkable achievements of mathematical physics to systems that match its models only formally lead to complicated theories that are difficult to visualize as a whole, just as attempts to extend the geometry of second-order surfaces and the algebra of quadratic forms to objects of higher degrees quickly leads to the detritus of algebraic geometry with its discouraging hierarchy of complicated degeneracies and answers that can be computed only theoretically.

The situation is even worse in the theory of partial differential equations: here the difficulties of commutative algebraic geometry are inextricably bound up with noncommutative differential algebra, in addition to which the topological and analytic problems that arise are profoundly nontrivial.

At the same time, general physical principles and also general concepts such as energy, the variational principle, Huygens' principle, the Lagrangian, the Legendre transformation, the Hamiltonian, eigenvalues and eigenfunctions, wave-particle duality, dispersion relations, and fundamental solutions interact elegantly in numerous highly important problems of mathematical physics. The study of these problems motivated the development of large areas of mathematics such as the theory of Fourier series and integrals, functional analysis, algebraic geometry, symplectic and contact topology, the theory of asymptotics of integrals, microlocal analysis, the index theory of (pseudo-)differential operators, and so forth.

Familiarity with these fundamental mathematical ideas is, in my view, absolutely essential for every working mathematician. The exclusion of them from the university mathematical curriculum, which has occurred and continues to occur in many Western universities under the influence of the axiomaticist/scholastics (who know nothing about applications and have no desire to know anything except the "abstract nonsense" of the algebraists) seems to me to be an extremely dangerous consequence of Bourbakization of both mathematics and its teaching. The effort to destroy this unnecessary scholastic pseudoscience is a natural and proper reaction of society (including scientific society) to the irresponsible and self-destructive aggressiveness of the "superpure" mathematicians educated in the spirit of Hardy and Bourbaki.

The author of this very short course of lectures has attempted to make students of mathematics with minimal knowledge (linear algebra and the basics of analysis, including ordinary differential equations) acquainted with a kaleidoscope of fundamental ideas of mathematics and physics. Instead of the principle of maximal generality that is usual in mathematical books the author has attempted to adhere to the principle of minimal generality, according to which every idea should first be clearly understood in the simplest situation; only then can the method developed be extended to more complicated cases.

Although it is usually simpler to prove a general fact than to prove numerous special cases of it, for a student the content of a mathematical theory is never larger than the set of examples that are thoroughly understood. That is why it is examples and ideas, rather than general theorems and axioms, that form the basis of this book. The examination problems at the end of the course form an essential part of it.

Particular attention has been devoted to the interaction of the subject with other areas of mathematics: the geometry of manifolds, symplectic and contact geometry, complex analysis, calculus of variations, and topology. The author has aimed at a student who is eager to learn, but hopes that through this book even professional mathematicians in other specialties can become acquainted with the basic and therefore simple ideas of mathematical physics and the theory of partial differential equations.

The present course of lectures was delivered to third-year students in the Mathematical College of the Independent University of Moscow during the fall semester of the 1994/1995 academic year, Lectures 4 and 5 having been delivered by Yu. S. Il'yashenko and Lecture 8 by A. G. Khovanskiĭ. All the lectures were written up by V. M. Imaĭkin, and the assembled lectures were then revised by the author. The author is deeply grateful to all of them.

The first edition of this course appeared in 1995, published by the press of the Mathematical College of the Independent University of Moscow. A number of additions and corrections have been made in the present edition.